PICTURES OF THOUGHT:
THE REPRESENTATIONAL FUNCTION OF VISUAL MODELS

A Thesis in
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by
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1. Models and representation

1.1. Pictures and language

The concept of representation plays a crucial role in our explanations, argumentation, and descriptions. One entity stands for another, replaces another, substitutes itself for another—represents it. Alternatively, an image or a picture represents a state of affairs in the world, renders it in the visual domain: we represent a situation visually. What does visual representation accomplish?

The idea of representation is malleable, pliant, rich, multiple, terrorizing, and endlessly frustrating. The concept of representation functions as a relation or a link between our perceptions and the world, between the pictures and forms we create and these perceptions, between our language and our perception, and between our pictures and language. The pictures, images, sketches and diagrams we produce of the world represent the world, link us to it visually. How do visual models function within theories, and how do theories describe the world independently of visual models?

The semantic connotation of the word "representation" is that of the German "Abbildung," which literally means
"imaging" or "picturing." Moreover, Abbildung contains the prefix "Ab-" meaning "from" or "away from"; an Abbildung is thus an image "from" or "away from" reality.

The dominance of the study of language in much of twentieth-century philosophy has made thinkers look to linguistic tropes to explain the representational function of models. The two major tropes to compare and contrast words or ideas are metaphor and analogy. There are important differences between these two tropes: it is possible to give a clear, even formal definition of analogy; but it is much harder to characterize metaphor, especially when one does not want to appeal to analogy in the process.

The notion of analogy is not limited to the realm of language, but metaphor seems to depend on the possibility of distinguishing literal from figurative language.

The traditional definition of analogy comes from Pythagoras through Plato and Aristotle: A is to Y as B is to Y, as C is to Y, and so on. It is immediately clear that analogy is not a comparison between two things, but between two relations holding between things. The concept of analogy plays a fundamental role in the definition of many other concepts used in the study of representation: isomorphism, parallelism, similarity, resemblance, substitution, even metaphor. The concept of substitution, in particular, is the key to understanding a widely-held description of the representational function of visual
models as a didactic device to avoid the complexity or mathematical intricacy of a theory: the visual model is a low-level substitute for the linguistic and mathematic descriptions of the theory. The concept of analogy also relies on the notion of correspondence, which has a long and tormented philosophical history and far-reaching ontological and metaphysical implications.

A "metaphormania" has gripped philosophers who have come up with studies and theses to explain the role of metaphor in language (Johnson 1981, ix). In a seminal paper, Max Black argues that metaphor should not be reducible to analogy. He explains that we do not capture the exciting possibilities of metaphor by merely explaining metaphor as the substitution of one set of terms for a more literal set of terms that could just as well have been used.

Instead, he proposes the interaction view of metaphor: two structures are contrasted, and the interaction between them produces the metaphorical effect (Black 1962, 31).

Trying to define the concept of metaphor without some reference to language, and specifically to the distinction between figurative and literal use of language is rather difficult. The notion of metaphor can be adapted or stretched to cover cases in the visual arts (Goodman 1976), music (Hatten 1993), or architecture, but this shift away from the linguistic domain always makes the application of the notion of metaphor more problematic. The application of
metaphor to non-linguistic realms can inform the study of representation in these fields, but it will not clarify metaphor itself. The concept of interactive metaphor will help us to conceive initially of the notion of a generative visual model. The productive aspect of the metaphor, which results from the interaction of two domains (structures), can fruitfully be compared with the generative aspect of visual models. Max Black (1962), Rom Harré (1960), and Carl Hausman (1993) have described the representational function of models using the notion of metaphor.

I do not claim that a visual model represents as a metaphor because a metaphor itself does not represent anything. The interaction view of metaphor requires two structures to interact, and rests on the distinction between literal and metaphorical use; this makes it difficult to think of models as metaphors.

1.2. Models and representation

Most of the contemporary philosophical discussion of representation takes place in aesthetics. Representation is the relation between a picture or image and "reality," or whatever is to be represented, be it an object, person or landscape. Two theories are currently in vogue: one holds that representation is essentially mimetic while the other,
based on semiotic theory, proposes that a picture represents and refers as a (linguistic) sign.

The theory of pictorial mimesis can be traced back at least to Plato's description of the painter as someone who "holds a mirror up to nature" (Blinder 1986, 19). The representational function of mimesis can be defined as similarity or resemblance. On this theory, a picture represents reality because of its similarity or resemblance to (perceived) reality; the picture is a copy or image (eidos) of reality. The theory of pictorial mimesis has been criticized on many counts. Both similarity and resemblance are symmetrical relations, but representation is not. The reality that is being depicted may resemble or be similar to the picture just as the picture resembles or is similar to the reality, but the reality does not represent the picture.\footnote{Blinder claims that these criticisms are a "misrepresentation" of pictorial mimesis, but I believe that the frequent appeal to unexplained notions of resemblance and similarity by adherents of the mimetic theory and others underscores the importance of these criticisms (Blinder 1986, 19-20; Goodman 1976, 4-5).} Further, we can have representation of one entity by another without similarity or resemblance, and similarity or resemblance between two entities without representation. Finally, similarity or resemblance is a relation between two entities that are being compared, whereas the thing represented is the thing in reality itself. "I see Eisenhower when I see his picture, I do not
see someone or something that looks like Eisenhower" (Sokolowski 1977, 5).

The above is a description of the "naive" theory of pictorial mimesis which is still implicitly or explicitly held by a large number of thinkers. There is also another theory of pictorial mimesis, one which harkens back to the connotations of imitation as parody or mimicry contained in the Greek meaning of mimesis; these are predominantly evident in the theater. Here the picture or image is a deliberate and self-conscious distortion of the reality being represented, paradoxically making fun of and paying homage to the Gods through the persona on stage (Sakabe 1985, 97). In mimesis as parody there is subversion inherent in emulation, and a dynamic interplay between copy and difference. This notion of mimesis requires a perception of difference sufficient to establish the parodic.

Moving away from the concept of similarity or resemblance towards the notion of (creative) difference is of great philosophical significance. Transposed to the realm of pictorial representation, this would mean that pictures represent by being simultaneously caricature and idealization. Visual models are both: the representation of the system is a caricature of the complexity of the system, but it also an idealization of that system. On this theory, representation does not consist solely in faithful
copying, but in the interplay between caricature and idealization created by distance and difference.

The "semiotic" or "semantic" theory of representation is a comparatively recent development, championed by Nelson Goodman and others. These authors take pictures to represent because they are signs or symbols in the way that words in language are. Pictures are to be conceived as "graphic symbols" (Maynard 1972, 243-44). This theory provides a concept of denotation as well as representation; it does not rely on the symmetrical notion of similarity but on the unidirectional concepts of signification or symbolization; and it can readily account for representations which are non-visual or non-realistic, such as music, abstract painting, or abstract sculpture, as well as all kinds of formal and informal languages. The troubling aspect of the semiotic theory of representation is that it is indeed far removed from a relation of picturing, and independent of the use of specific images. Words can function as signs within a language, and represent the objects they pick out by convention. The relation of images and pictures to what they represent is drastically impoverished if it is taken to consist merely in the assignation of a symbol to an entity in the world. The symbol is initially arbitrary, and represents by convention, before it acquires its own symbolic weight.
The concepts of representation found in aesthetics cannot explain the role of visual models in sciences, specifically those models which generate their own worldview and thereby influence theories. Representation as mimesis is not going to be of much use when there is no comparison between the images or perception involved. While the symbolic theory of representation could help explain how mathematical language represents either elements in the world or data, it does not explain how the pictorial nature of images represents.

A visual model is somehow less than a copy and more than a sign—or beyond a copy and not yet a sign. The visual models studied here exceed what can be perceived and redefine the visible, while representing and rendering accessible reality.

How do images and pictures represent a world by functioning as visual models for a certain state of affairs? The representational relation that we are focusing on links a body of linguistic or mathematical statements to the world through a set of images or pictures. The visual models thus exemplify a particular representational relation. A formal relation between visual model and theory is needed to explain why a visual model represents a particular theory. We will develop a projection function to account for a fundamental part of this formal relation. The image as
projection is only a proto-picture; a series of graphic manipulations are required to turn the projected proto-picture into an image. The process often demands a "visual leap of faith" for us to assimilate the resulting image as a model for a theory.

1.3. Models in the philosophy of science

Historical studies in the philosophy of science chart the changes in attitude towards scientific models, their development in the work of scientists, and the multiplicity of their uses. Scientists from a wide range of disciplines have reflected upon the use of models in their scientific practices and have elaborated more general and abstract theories of modelling.

Models operate at the intersection between the knower and the known, or between the represented reality and the perceived or measured reality. The design, choice and use of these models thus reflects fundamental features of reality, cognitive faculties, and epistemic capacities. Models are powerful tools in our description of reality and in the body of knowledge that seeks to describe and interpret this reality. The term "model" is one of the most overused concepts, both in the philosophy of science and in everyday language; it can be used for a truly kaleidoscopic array of entities. Nelson Goodman writes:
Few terms are used in popular and scientific discourse more promiscuously than "model". A model is something to be admired and emulated, a pattern, a case in point, a type, a prototype, a specimen, a mock-up, a mathematical description - almost anything from a naked blonde to a quadratic equation - and may bear to what it models almost any relation of symbolization. (Goodman 1976, 171)

The proposed concept of representation, although limited to a particular subset of models, informs the study of modelling.\(^2\) Understanding the function of this class of models is only one of the aims of the work; insights into the mechanism of representation and the epistemological function of pictures will also emerge.

There is a widespread use of the word "model" to describe human cognitive processes. Models of perception, of what and how we perceive, are being studied by philosophers, psychologists, and cognitive scientists working in Artificial Intelligence (AI). Models of perception and cognition constitute the lion’s share of research on models by philosophers in the last four decades. Researchers in the philosophy of mind and the

\(^2\) In a private discussion, James Hoffmann warned me about the infinite task of evaluating all models; according to him, the best that can be done is to evaluate models in each specific setting, a task he is pursuing (Hoffmann 1990; J. Hoffmann and P. Hoffmann 1992). His tactic appears to me just as infinite, and needlessly coy; new relations of modelling that are discovered or invented enrich the evaluation of all models.
interdisciplinary program of cognitive science analyze, construct and test models of human perception. They investigate how the human brain forms images (which are called internal models) of external reality, how these internals models of reality relate to varying conditions of perception, and how these models can be codified into computer programs. I will not study these kinds of perceptual models here. Although these are necessary visual models, they are not models of a theory independent of perception. I am not concerned with the perceptual aspect of representation: I seek to comprehend the epistemological aspect of images as such without regard for the way our brain produces them. The visual models that I analyze in my case studies are not models of the perceived external world but models of collected data appropriated by theories. The kind of visual model I analyze is an image or picture and not a brain-function which enables us to make sense of what we see. Although I share the name "visual model" with cognitive science and the philosophy of mind, we are dealing with radically different entities.

The term "model" derives from modulus, a diminutive of modus, the Latin for "measure," but its meaning has evolved to include, among others, a scale reproduction, an analogical system, or a projection model. Most thinkers who have pondered the role of models advance their own
classificatory system for them, but they also readily acknowledge that rigid classifications are arbitrary and confusing, and that categorization is pragmatic at best.

Some useful theoretic and practical distinctions can still be made. The model as scale reproduction operates in a comparatively narrow epistemological field, since both the model and the system being modelled share many important and specifically concrete features; they differ in abstract features such as size or degree of complexity. Some scale reproductions, such as scale models of steam engines, represent the inner constitution of a mechanism; others, such as architectural scale models or some anatomical models, only represent the visible outside shape. The model as scale reproduction is described in terms of the usual opposition between essential and non-essential features: the essential feature can be either function or appearance. In the case of the steam engine, the scale model reproduces the function of the system; in the case of the architectural model, it reproduces the appearance.

We also find this distinction between function and appearance in the essential properties of visual models. A diagram represents the function of a system without necessarily sharing its appearance. A sketch of a system seeks to reproduce the appearance of the system, often without regard for its function.
The model conceived as an analogical system functions as an independent system by analogy with a theory in a different epistemological realm. The analogical model is not restricted as to its shape or kind: for example, a theory about reproduction can function as a model for the theory of evolution (Ruse 1973a; Ruse 1973b); billiard balls become models for kinetic theory (Hesse 1966); real, wet waves model both acoustic and electric waves (Kargon 1969).

The criteria for the choice of the model are the strength and relevance of the analogy. The theory is known and studied before a model is chosen for it, and is "epistemologically prior" to the model that illustrate it. In the case of the analogical model, the dynamic of modelling comes from the theory and reaches "outward" towards the model.

A projection model is constructed by geometrically projecting the data onto a grid to produce an image; it generalizes data into a visual structure. Projection models are part of both the data and the theory. Maps, pictures obtained through graphical imaging of processes, fractal images, but also economic models and models predicting large-scale occurrences such as climatic or astronomic developments, are projection models. Projection models have recently gained in importance through the rapid development of computer simulations. Projection models are epistemologically prior to the theories in which they
function, and the dynamic of modelling works outward from the model to the theory. This kind of model shows the potential that constructed pictures can have for generating theories, and thus for furthering scientific understanding.

1.4. Visual models

A visual model is a representation in visual form of an empirical system as conceived by a theory. The information it represents may or may not be duplicable by language, a mathematical formalism, or another model. A theory is knowledge of aspects of reality gained through research and codified in a language or mathematical formalism utilizing a set of data. The set of data that makes up the empirical system includes a theory about what is being measured and how it is being recorded. A primarily visual model cannot be totally duplicated by language or a mathematical formalism, although it may be duplicated by another visual model. A visual model is necessary if some theoretical claims require the use of images. I often use the word "image" or "picture" to refer to the visual model as an image separated from its relation to the theory. The term "image" connotes something immediately given, whereas "picture" connotes some construction or sketching.
There are many classifications of visual models. Harry Robin divides scientific images into six categories: observation, induction, methodology, self-illustrating phenomena, classification, and conceptualization (Robin 1992). Uwe Porksen distinguishes three main categories of visual models: for example drawings produced by hand, mechanoscopy (scanning devices, sonars, but also photography), and graphical representation of numerical data (Porksen 1992, 2). This subdivision separates visual models according to function, which is useful for charting the reliance on mechanical devices and the "new objectivity" produced by mechanoscopy (Porksen 1992, 8-11). But some visual models function in all three of Porksen's categories: mechanical scans of an empirical system, enhanced by hand (image manipulation), which graphically represent numerical data. I shall refrain from classifying visual models, because I wish to stress their multiple functions, and the different levels at which they can function as images, graphs, sketches of perception, and so on.

Images are not just pictures in the mind's eye, but have functioned as developmental symbols. The fascination with images and the literally fabulous role they have played in the elaboration of visual models can be illustrated by some famous examples. The German chemist Kekulé had, on his own account, a dream of a serpent eating its tail; this image led him to develop the visual model of the benzene
ring. When the Dutch chemist Van 't Hoff was studying the valence properties of chemical compounds, he came up with a tetrahedral model which he had constructed in his imagination without deducing it from experiment. Van 't Hoff got roundly criticized by colleagues for his unscientific method, but he defended his aesthetically-motivated inspiration and was later vindicated by experimental results (Root-Bernstein 1985, 50). The reasons for the adoption of a model, like the reasons for preferring some theories, are often aesthetic. Niels Bohr patterned his model for the structure of the hydrogen atom on the solar system, but this visual model would never have been adopted had it not also allowed for an interpretation of atomic theory in which the orbits of the planets corresponded to quanta. Indeed, if any visual model leads to incorrect assumptions or results, it will soon be altered, dropped or replaced. It may be difficult to disentangle our projection of scientific theories onto visual models from our aesthetic appreciation of certain images. Images may also be preferred for their symbolic, quasi-religious or mythical implications.

The tree diagram illustrates the variety of different functions of representation that a visual model can acquire. The tree diagram as the guiding model for the charting of species is the only image in Darwin’s _Origin of the Species_; it already represented a paradigm of taxonomy. There are important metaphysical connotations of the tree-diagram: a
single species "at the top" of a strict hierarchy, evolution which occurs through branching, no cross-connection or cross-fertilization between species at the same level but on different branches, and the emphasis on lineage.

After Darwin, the tree-diagram began an illustrious career in taxonomy. The extent to which diagram and image are intertwined is striking. The visual model as classificatory system (tree-diagram) doubles up as a visual model as image (drawings of natural trees); this is shown rather well by Robert O'Hara in his evaluation of tree-diagrams in ornithology (O'Hara 1988; O'Hara 1991; also Blum 1991). Figure 1.1 shows the first diagrammatic sketch of a tree-diagram from Darwin's notebooks. Figure 1.2 shows a more elaborate tree-diagram by drawn by Ernst Haeckel. Figures 1.3 shows a vertical representation of the evolutionary tree of birds by Furbringer, while figure 1.4 shows a horizontal projection of the same tree. There is a continuous development from diagram to image here; the tree as symbol also lurks in the background as a metaphysical assumption. The image of a tree embodies a hierarchical order designed to classify and name an ever-increasing variety of species.

Another example of the interdependence between classification and diagrams is the periodic table of chemical elements, first drawn up by Mendeleev. The periodic table charts the course of all future research on
Figure 1.1. Tree-diagram from Darwin's notebooks. From Robin 1992, 160.
Figure 1.2. Tree-diagram by Haeckel. From Robin 1992, 161.
Figure 1.3. Furbringer's evolutionary tree of birds. From O'Hara 1991, 267.
Figure 1.4. Horizontal projection of Furbringer's evolutionary tree. From O'Hara 1991, 268.
chemical elements, since it classifies and thereby defines an element by its atomic number. The remarkable regularity of the first 130 elements has prompted scientists to continue the search for the next elements, to the point where they create ephemeral elements lasting fractions of a second under tight laboratory conditions. The line between discovery and invention, although it is not clearly drawn, has definitely been crossed here: the elements are constructed to fit the theory.

1.5. Models, theories, and reality

The simplest and most naive relation between the discourse of science and the object of its study yields the following diagram:

\[
\text{theory} \quad \longleftrightarrow \quad \text{reality}
\]

Figure 1.5. The relation of theory and reality.

Let us separate visual models from the theory in which they are articulated, and replace the controversial term "reality" by the concept "empirical system." By "empirical system" we will mean the data analyzed through observation or the use of machines, either in a natural setting or in a laboratory-controlled experiment. We now have:
Here, both the visual model and the theory are related to the empirical system, which they explain, exemplify, or illustrate. The analogy between the visual model and the theory accounts for possible equivalences between their respective relations with the empirical system, which allow us to substitute one for the other.

By rotating the picture, we can show the visual model as only one of the many possible ways in which the theory is related to the empirical system:

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According to Downes, the relation "=?=" is usually thought to consist of isomorphism or similarity, a notion he succeeds in discrediting (see chapter 2, sections 2.1 and 2.2). But whatever "=?=" is, it is different from "==>," which holds between models and theories; we are still confronting two different relations.

We are misled by the pictorial nature of the diagram into according the visual model a distinct identity, with an ontological status distinct and on the same level as both the empirical system and the theory. But the visual model should be considered as a relation of representation between the theory and the empirical system, and not as an ontologically distinct entity:

\[
\text{theory} \quad \langle-------- \text{visual model} \quad ----> \text{empirical system} \\
\text{representation}
\]

Figure 1.9. The visual model as a relation.
We can expand our diagram to provide for other possible mediations between theory and empirical system:

```
visual models

theory ----> mathematical models (equations) ----> empirical system

linguistic description

??? - other models

representation
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Figure 1.10. The visual model as one relation among others.

This revised diagram denies a distinct ontological status not only to visual models, but also to mathematical and linguistic models. The models make up the theory, the theory is composed of models; the theory is nothing more than the conceptual unity of these models. The empirical system as defined by a theory consists only of the various models. It now seems as though only the models exist, and that both theory and empirical system are constituted by them: we have only representation. We can claim either that only representation exists, or that both theory and the empirical system are distinct entities.

A distinction has usually been made between that which models (active) and that which is being modelled (passive), hence the arrow in Downes' "===>" relation. But here
language fails or entraps us: we say the model models the phenomena (the empirical system), and that it is a model for the theory. Or, we could say that the theory models the phenomena by means of the model; or again, that the modelling relations of model and theory are analogous. For logicians working on model-theory, a model "for an axiomatized theory is an ordered set in which the postulates of the theory are satisfied" (Achinstein 1964, 329).

Our everyday and technical usage of language does not produce a clearly defined use of the terms "model" and "modelling" from which we could derive the relations of modelling holding between different entities; the philosophy of language's effort to shackle understanding to linguistic usage is thwarted here. Modelling is an activity; the distinction between active and passive is of secondary importance.
2. Maps and projection

Projection consists in turning numbers into images. Initially, this is a problem of topology and geometry, of defining a topological space, and projecting numerical values along chosen axes. Once a shape has been defined in a topological space, it can be transformed by simple mathematical manipulations such as rotation or scaling, or by projections from one dimension to another, for example from the surface of a sphere to that of a plane. These projections, or the relation holding between any two structures, can be homomorphisms or isomorphisms. Similarity projections are projections which preserve certain features of the shape, such as angles or measures.

These permutations and their names have precise mathematical definitions, which need to be contrasted with the use of the same terms as notions in philosophical and everyday language. The notion of similarity or resemblance in everyday language cannot be derived from the mathematical notions of isomorphism or similarity without broad and unwarranted metaphysical assumptions. The conceptual basis for the notion of similarity lies in identity and difference, especially when taken as elements of a more holistic notion such as general resemblance.
The production of projection models initially requires projection conventions, but it also involves manipulations of the image from the raw projection model. Manipulations of the constructed picture include the use of symbols, color, three-dimensional simulation, shading, animation, and so on. These manipulations have enormous influence on the finished image, and both care and imagination is needed to combine them with projection permutations without engendering confusion.

Maps are a good first example of projection models, since they are enormously varied, well-known, and often confused with images derived from perception. Most of the other visual models studied in the next few chapters are essentially maps, which codify knowledge about a system in visual form through projections and manipulations. These sophisticated maps advance our sense of the visible beyond the bounds of what can be perceived and redefine the kind of knowledge of the world that can be gained through visualization.

2.1. Isomorphism, similarity and resemblance

Many theories of representation or modelling make use of the mathematical concept of isomorphism, or the kindred notion of homomorphism. I shall analyze the mathematical
concepts involved, distill the concept of projection from this analysis, and show what relations occur between these formal concepts and the more vague notions such as similarity and resemblance. The view that models are analogies and the mimetic theory of representation both appeal to similarity and resemblance.

Isomorphisms and homomorphisms are two special cases of morphisms, or formal projections, defined in mathematics. Morphisms are projection relations between two structures; a homomorphism connects every point from system A to system B, without, however, connecting every point of structure B to structure A. An isomorphism is a symmetrical relation; it connects every point from system A to every point of system B and vice-versa (Frey 1969).

A morphism is a formal, and hence neutral, relation between two structures; it merely projects elements from one structure onto another. If it can be established that two structures are isomorphic or homomorphic in some domain or some respect, this would define the relation far more exactly and precisely than defining the one as the model of the other. If two structures are isomorphic, it is always possible to replace or substitute the one for the other; if one structure is a model of something, then the other is also a model for it. These features account for the popularity of isomorphism among philosophers eager to
explain modelling, analogies or similarity. In fact, the application of isomorphism is practically restricted to a formal mathematical or geometrical domain. The structures between which an isomorphism holds must either be both known completely, or one must be defined in terms of the other; this should never occur in the setting of a scientific research program.

The concept of isomorphism is a substitute or front for the concept of correspondence, which has a long and troubled philosophical history; isomorphism is a special case of correspondence. All the problems of dualism and of defining the ontological realms between which there is a correspondence also apply to the concept of isomorphism. These thorny problems are obscured by the ease of application of this formal concept in the mathematical realm. But isomorphism is a special case of correspondence. The use of the term "isomorphism" suggests that the reasons for the correspondence relation are both known and defined, which is rarely the case. The situation deteriorates even further when similarity and resemblance are taken to be a kind of isomorphism.

Similarity also has both a mathematical and a common-sense meaning. In mathematics, a similarity is a transformation which preserves specific properties, such as angles or measures. The notion of similarity in everyday
language is rather different; the notions of resemblance or likeness are its close synonyms. Structures, objects, weather patterns, historical occurrences and peoples' personalities can be said to be similar. To call things similar is to claim that there are respects in which they are the same and respects in which they are different. This is very close to the difference between the positive and negative aspects of analogy, and one reason for the kinship of analogy and similarity. But the knowledge necessary to distinguish the identity and difference among the various properties of the two entities which are called similar must already be possessed by both speaker and listener, or at least tacitly implied. We conceive of a "general resemblance," where there need not be any specific properties that are identical and different, but where one entity "as a whole" is taken to be similar to another "as a whole;" the resemblance need not rest on clearly shared attributes. Insofar as the notion of general resemblance blurs specific similarities in the interest of an overall impression, it is even further removed from the formal relation of isomorphism.

"Formal similarity" in the mathematical sense is a valuable tool for describing the properties of projection functions. But in everyday language, attributing similarity is a value judgement based on comparison, and is moreover a vague and intuitive notion.
It is difficult to see how one could derive the notion of similarity from the concept of isomorphism, as so many philosophers writing about the representational function of models have done (Dambska 1969; Forge 1983; Giere 1988). An isomorphism is a symmetrical relation of formal projection between two structures, while similarity is a loosely defined relation between two elements or an attribute of a group of elements. We say, for example, that entity A is similar to entity B, or that entities $E_1, \ldots, E_n$ are similar.

If there is a formal isomorphism between a set of axioms and the geometrical definition of a triangle (which can yield the image of a triangle according to a set of depiction conventions), there is no implication at all that either one axiom is similar to a property of the triangle or that the delimited set of axioms and corresponding theorems is similar as a whole to the geometrical definition of a triangle. Since the two structures operate in two exclusive realms, they are not similar. The formal relation between them is an isomorphism, and the same formal properties can be attributed to the two structures. The only value-judgement relation that we can attribute to these two structures is, in some specific cases, equality or equivalence.

To attribute similarity to a relation or a number of entities is not to define a formal relation, but to make a
value-judgement about them; similarity is an evaluation based on comparison. The statement of similarity is intended with both a pragmatic and productive effect in mind. A pragmatic effect is intended because by noting or accentuating the similarity I do not explain or even clarify the formal relation between the elements I am comparing. A productive effect is intended because I have a certain outcome in mind. By noting the similarity between these elements, I add something to the already existing formal relations between them. The judgement of similarity is thus extrinsic to the possible formal relation between the elements in question. Noting a similarity is presenting the elements involved in a new light: it is a description rather than an explanation. By contrast, morphic relations between two structures are intrinsic: although pointing out or noting these formal relations constitutes an advance of thought, the projection relation of isomorphism or homomorphism is contained in the formal structures themselves. There is no value judgement involved, nor is there formally speaking anything added to the relation between these structures by noting the specific property of the relation.

The value-judgement and comparison inherent in the claim of similarity make the concept rest on an intuitive apprehension that a comparison can be made in the first place. When I say: these two people have similar
personalities, the reason for comparing them is an intuitive sense that there may be something similar or comparable about them. I do not start searching the world for entities to compare in the hope that there may be something similar about them. The reasons given for the judgement of similarity always come after the fact, after the judgement. The personalities of these two people strike me as similar; to explain my judgement in this case, I will give reasons. But I do not first compare and tabulate the attributes of the two personalities and subsequently pronounce a judgement of similarity.

The intuitive aspect of similarity is a not a defect, nor should it stop us from noting and using similarities. But it limits the explicative power of similarity to account for modelling or representation, since noting similarity does not consist of advancing reasons for this value-judgement.

It is not for the philosopher to legislate the use of language, and in everyday situations and language few complications are likely to result from claims about the similarity of things. No one will ask: what exactly is it that you have in mind when you say that these things are similar? But we must become clear about what kind of representational function visual models have; this involves separating well-defined formal procedures from intuitive hunches which may make for surprising correlations, but do
not explain the role of visual models in scientific research.

2.2. Projection and manipulation

Projections are laws or equations applied to a topological space which can be encoded in mathematical symbols; they determine the shift from numbers to images through the application of geometry. Manipulations are transformations of the internal structure of the image; they include colors, scale, but also three-dimensional perspective, the use of various symbols, and more recently, artificial light source and shading effects. While the projection function determines the representational function of the visual model, manipulations play a crucial role in the way the picture "looks," how it appears to us, how it affects us psychologically, and thus how we can get information from it. There are an infinite number of ways of constructing a picture according to a single projection function, even though these possibilities are usually restricted by applicable conventions. There are likewise a great variety of choices involved in the manipulations of the picture. While these may seem arbitrary from a formal
point of view, they have enormous impact on the visual quality of the picture, and hence on its applications.

Visual models as images operate mostly in a two-dimensional space delineated by chalk on blackboard, pencil on paper, or graphic image on computer screen. But, since human beings operate in what they perceive as a three-dimensional environment, we have developed pictorial conventions for representing three-dimensional objects (solids) on a two-dimensional surface (flats). This pictorial technique is both a projection and a manipulation, used to produce an intentional illusion to which we must subscribe to see a (pseudo) three-dimensional figure on a flat surface.

The "front" panel of this cube could be either ABCD or EFGH, depending on the gestalt shift of the perceiver. This also means that at least these two different cubes are represented here.

Figure 2.1. Three-dimensional perception on a plane.

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3The importance of the design of visual representations for their effectiveness is well illustrated by Edward Tufte in Envisioning Information, which showcases several design techniques. The author presents optimal design solutions for representational problems with a fixed number of variables, such as a train timetable. Although the visual models I analyze represent "speculative" information derived from theories or data, these manipulations and pictorial conventions also apply to them (Tufte 1990).
We use the representational conventions of the two-dimensional figure to point to, suggest, or conjure up another, three-dimensional figure. The possibilities and limitations of these pictorial manipulations of natural geometrical dimensions are strikingly illustrated by M. C. Escher’s lithographs. This technique need not be limited to the illusion of three-dimensional solids, but can be used for many other purposes; it can be used to construct a virtual three-dimensional image of data which has no geometrical dimension at all.

The impressive result of three-dimensional projections used as manipulations of a figure is aptly illustrated in figure 3.3. The colored surface in this figure is an isoplane, a two-dimensional cut of the values of a variable on the surface of the plane. A color scheme based on the rainbow indicates the intensity of the variables, from violet for the lowest to red for the highest values. The use of color is reinforced when the values of the variables are projected in three dimensions, producing red peaks and violet troughs. There are two projections into three dimensions: one for the variables already in three dimensions, the other for the variables of the isoplane. While these projections make use of the same three-dimensional space, they indicate two different dimensions of data. This technique can constitute an excellent use of the representational properties of visual space, but is also
misleading because of the ingrained nature of pictorial conventions. The creative use of pictorial conventions already shows that a picture does not have to "look like" a thing in order to represent it.

A picture has its own inner systematic coherence, in which the choice and elaboration of symbols plays a vital role. The use of symbols is initially arbitrary; then the legend fixes them univocally. But the symbols themselves can become part of the representational function. This occurs in musical notation, where the symbols that were initially part of the system direct and restrict possible symbolic elaborations. A "symbolic transference" of the initial symbols into the shape and structure of the new symbols takes place. This limits the ability of the symbolic system as a whole to respond to historical pressures to extend or change musical notation (Benammar 1990). Classical musical notation is not just a visual projection model but a hybrid in which symbols have both a symbolic and representational function.

Color has been so intimately intertwined with representational beauty that it is rarely used as a separate dimension of information from the more prosaic gray shading, although it does make for far superior contrast. Color carries information by pictorial convention: red means hot or interdiction, blue means cold or water, green means "go" or indicates lowlands on geographical maps. The possible
uses of colors are restricted by the culturally-shaped understanding of its representational function and the limitation of conventions.

Manipulating the image involves negotiating the pictorial conventions and responding to the dictates of the cultural environment. We "see" a representation as a picture because of the cultural environment in which we operate, and decode the manipulations involved. Although cultural dependence contributes to the complexity of information that can be transmitted in a picture, it also restricts the use of variant or novel manipulations.

New manipulations techniques have become available with the extension of graphical imaging to the computer screen. The use of shadows and the shading of shapes according to a virtual light source enhances the illusion of three dimensions on a flat screen. These techniques are not usually manipulations representing specific variables, but they enhance the aesthetic quality of the image. This aesthetic quality influences the reception and comprehension of the picture.

It is always helpful to determine the data-dimension of a projection. Doing so gives an immediate schematic overview of the complexity of the projection function (mapping projection) that is used. Projection along the axes of the three "natural" geometrical dimensions, manipulation techniques such as coloring, and icons or
symbols may all be used represent the values of one variable. The number of data dimensions will usually be determined by the number of variables to be represented. One variable can be represented by more than one data-dimension, as in the case we discussed above of the isoplane projected as a three-dimensional image in figure 3.3. It is also possible, and typical, that several variables are conflated into a single data-dimension. The data-dimension should also indicate in which respects the projection is only partial, and the role played by manipulations in the picturing process. Finally, the assignment of variables to projection functions or manipulations gives at least some indication of the priority accorded to variables, and therefore of the intended role or aim of the representation.

2.3. Maps and mapping

A map is the result of a process of mapping, a visual model, and the representation of empirical data. Geographical maps are visual models which represent specific features of our environment according to theories of cartography. Mapping means representing data in visual forms such as diagrams or images; but it can also describe a homomorphic relation informally, when we say that we "map something" onto something else. The sciences also use a
more specialized sense of the word "map" which refers to the projection function, the series of geometrical and topological transformations that determine the construction of a visual image from a set of data. Mental maps are mental models, and not part of the subject of this work.

Maps are visual models of cartographic data; without visual space, there is no map. In cartography, the use of measuring devices was supplanted by photogrammetry (the ability to take measurements from photographs), which has been revolutionized by satellite photography and new scanning methods (Makower 1986, 13-16). The use of maps is widespread both in everyday life and in scientific research.

We shall distinguish maps, which are based on projection, from images based on perception. The purpose of clarifying the distinction between maps and images is to make projection the constitutive feature of representation.

A geographical map is a visual representation of a section of territory on a specified scale, and employs representational conventions for line drawing, colors and symbols. How do maps represent? What makes a map a representation of a territory? What is the formal relationship between the thing that represents and the thing represented?

Jorge Luis Borges provides us with the absurd limit of map-making: an emperor, fearful of losing control over his ever-expanding dominion, orders a team of cartographers to
create the perfect model of his empire, a full-scale map which faithfully records every detail of the territory. As the cartographers execute the imperial edict, however, the country they had set out to represent vanishes, displaced by the prodigious effort required to create its perfect model. The emperor retains only an immense and useless map, an empty representation of something that no longer exists.

One can imagine a full-scale map functioning as a perfect isomorphism between territory and representation. To call this product a map is misleading: a map is not a full-scale isomorphism, but a projection, which involves re-presentation and re-construction.

Geographical maps are projections in two dimensions and projections from three to two dimensions. Let us assume that we do not have to take into account the curvature of the earth. A city map is not drafted from any singular vantage point or perspective; rather, it is reconstructed from measurements. Today most maps are drafted with the help of aerial photographs, yet the map is not the bird's-eye view either: specific mathematical permutations are required to achieve the non-perspectival representation which will conserve the correct distances on the ground (Monmonier 1991). A map is a schematic diagram, a pictorial representation of distances on the ground.

A map is not a schematically simplified photograph: it does not have perspective. A map is a reproduction and not
an image. Because so many of us have accepted the idea that the maps we use in daily life are images, pictures conceivably taken from balloons floating over the city, we confuse reproduction and images. It is true that an aerial photograph of a city taken from sufficient height would almost match a reconstructed map point for point. In skydiving, one's first impression of the view from above is that it is "like a map." Even specialists, however, occasionally slip: Stephen Hall claims that "no-one except a few astronauts have seen [the outline of the United States as drawn by Rand-McNally] in the "real" world" (Hall 1992, 19). But the perspectiveless cartographic rendering of the United States cannot be perceived from any vantage point.

Geographical maps are also projections from three to two dimensions. The cartographer can choose between several projection techniques according to the features of the three-dimensional surface that need to be retained or emphasized (Monmonier 1991). In the case of a map of the world projected from a three-dimensional model (a globe), the main features would be size, shape, and position (distances). The Mercator projection (named after the Flemish cartographer Gerard Kremer (1512-1594)), which is used for most world maps, conserves all angles on the surface of the globe and represents them on a plane without distortion, but to the detriment of both size and shape. The shortest distance between two points is not a straight
line but must follow the projection curve. In addition, the Mercator projection places the equator two-thirds of the way down, drastically reducing the size of the Southern Hemisphere; this is a politically sensitive point which has led the United Nations to adopt the Peters projection, which puts the equator in the middle. The image we have of the geographical entity "United States of America" is thus a fiction, a re-production that depends on the method of projection used. On the Mercator projection, the distortion increases as one approaches the poles; the shape and size of Greenland, for example, suffer a drastic amplification. The only way to represent both the shapes and the sizes of countries without distortion due to projection is to use a globe (Makower 1986, 12; Monmonier 1991).

The different maps that result from the projection process are competing but equally "correct" or "valid" representations of the world--that is, schematic diagrams of measurements, of distances on the ground. Pragmatic concerns determine the choice of a projection method and the resulting image, and there are as many possible images of a particular territory as there are needs to represent specific features of that territory. Maps are re-creations from a formalized transcription of data.
2.4. Maps and visualization

Mapping includes all efforts to visualize a territory, whether it be geographical, like the ocean floor; cosmological, like the structure of distant galaxies; or an inner part of the human body such as the brain. The new mapping procedures extend beyond perception and include Porksen's mechanoscopy, x-rays, positron emission tomography (PET) scans of brain activity, microwave altimeters to scan the ocean floor, and scanning tunnelling microscopes which can "see" at the molecular level (Hall 1992; 18-19, 78, 82, 242-43). Figures 2.2-6 show different maps created by mechanoscopic devices.

These mechanoscopic devices use non-visible wavelengths such as acoustic waves, and produce large amounts of data which typically need to be projected into graphical images with the help of a computer. The definition of the word "map," which used to be firmly attached to its geographical roots, has now been extended (and diluted) to "a graphic representation of the milieu" (Robinson and Petchenick, quoted in Hall 1992, 6).

The maps generated by the combination of mechanoscopic devices and computer graphics programs differ from more straightforward geographic maps in the complexity of the techniques involved in the recreation and reconstruction of a picture. While the transformation from aerial photographs
Figure 2.2. Microwave altimeter map generated from data gathered by the satellite Seaset. From Hall 1992, color insert.

Figure 2.3. Scanning tunnelling microscope. The structure of silicon. From Hall 1992, 242.
Figure 2.4. A combination PET and MRI (Magnetic Resonance Imaging) map of the brain. From Hall 1992, 150.

Figure 2.5. Computed tomography (CT) map of a plane of the brain. From Hall 1992, 147.
into surface maps using photogrammetry still makes use of images that could be perceived (though this is not necessary for accurate map-making), scanning techniques typically measure discrete values, and these series of numbers are transformed into pictures. Maps produced by scanning techniques are reconstructions from numbers, and can easily be stored and transmitted as numbers. Reconstructions from numerical data rely heavily on projection methods and a large number of manipulations, which greatly influences the pictorial aspect of the map. It may happen that two maps produced from the same numerical data will turn out completely differently, making it impossible to tell if they represent the same territory.

Despite the revolutionary technological advances spawning a whole new generation of maps and cartographic techniques, the mapping process remains one of probing the external world. Despite their sophistication and intricacy, scanning maps are still only visual models in a very limited sense, only minimally models. Although the representational relation of picture to the empirical system is more complex in the case of scanning maps than in the case of photography, their purpose is the same: to represent objects in the world.
The mechanoscopic human extends natural human capacities and metamorphoses into an alien with x-ray vision and magnetic-imaging sensors. The visual models that will be explored in chapters 3-5 extend the concept of the visible even further, since they represent empirical systems which could never be perceived by any conceivable means, and which are not obtained by passive probes of the external world. These visual models represent highly abstract phenomena which are difficult to grasp conceptually.
3. Models, graphical imaging, and animation

3.1. Projection models

The phenomena that scientific theories seek to explain have disappeared beyond the horizon of the visible because of their size or speed, or because they simply do not "exist out there" except as theoretical, virtual entities used in theories to explain other phenomena (Van Fraassen 1980). Beginning with the extension of the sense of sight through the telescope and microscope, the objects of scientific study have receded beyond immediate access. Now the scientific endeavor has evolved beyond scopia, the visual faculty, to the study of bubble chamber traces, background radiation, and presumed effects of certain particles on others. However, these non-visual phenomena leave traces which can be interpreted as data and projected as pictures. Hence the need for visualization through means of diagrams or visual models.

"Several of the ideas developed in this chapter come from discussions with a graduate student in Engineering at Penn State, Brian Moquin, who has developed an interactive computer system to study turbulence though graphical imaging. When the arguments or information derive from our discussion and are thus not totally my own, I have indicated this by the reference "(Moquin 1993)" at the end of the paragraph. The color plates 2.1 and 3.1 were developed using Moquin's system."
In a diagram, elements of the structure of the empirical system are represented in a simplified and schematic way. This can be done by sketching the structure of the system, or by plotting variables against each other geometrically. Graphical representation provides an overall perspective of the behavior of variables in the system and can reveal unexpected correlations between data. The graphic representation can thus modify the theory that accounts for this data. Diagrams are limited because they depict a particular perspective; while some information about the system is brought up in high relief, other information is often ignored or obscured.

Diagrams and sketches are not always simplified representations of a system. Jane Maienschein has shown how E. B. Wilson came to value the freedom provided by the representational possibilities of diagrams over photographs in the various editions of his seminal work, The Cell (Maienschein 1991). What is represented by a sketch is more realistic than photographs or images based on photographs. The reason for the success of the diagram or sketch lies not just in its didactic usefulness, but in its representation of a constructed reality interpreted by the scientist. The versatility and adaptability of diagrams and sketches accounts for their popularity as explicative aids, and is not an obstacle to their reliability as models.
Visual models are complex diagrams; they make use of our perceptive ability to distinguish three-dimensional shapes and colors and to interpret the information encoded.

A visual model of the empirical system is produced by projecting and manipulating data obtained by measurement of the phenomena. Presenting data graphically with a visual model is radically different from a mathematical presentation through equations or a linguistic presentation through a description. Graphical representation involves its own particular challenges, dangers and opportunities: by representing the system visually, new observations about, connections to, and explanations of the system are discovered.

Visual models represent measurements rather than objects that can be perceived; but these measurements do not necessarily constitute objects in the world, or guarantee any kind of reality. Indeed, the measurements sometimes only produce theoretical or virtual objects whose existence and qualities can only be postulated by the graphical representation itself. Yet the visual model represents something, even if that something is only a structure of virtual relations between numbers whose sole claim to "reality" rests on their being a figment of our scientific imagination.

As the complexity of the projection method increases, we lose all sense of a connection between our projected
image and something that could be perceived. The following diagram represents a continuum from mimetic copy to projected image (generative visual model), along which the visual models that we are investigating are situated:

- (non-artistic) - Geographical - Strange photographs maps attractors

COPY <--|----|------------|----|----------|----|--> PROJECTED
MIMESIS |                 |               |    IMAGE

- Illustrations - Turbulence - Fractal
- Architectural imaging images plans

- abstractness
LOW -------- - complexity of projection --------> HIGH
- dimension of representation

Figure 3.1. From copy to projected image.

This axis is supposed to suggest a difference in degree (of mimetic quality, or of "projectedness") between these various visual models. But there is also a difference in kind between an image that has a direct representational relation to another image or visual perception, as is the case for copies, photographs, illustrations, and some maps; and visual models that are constructions, which are not related to the visually perceptible properties of reality. Constructed visual models are only related to numbers, measurements or equations that are projected geometrically;
they are an extension beyond visual perception, representing beyond the visual horizon.

A projection model is not a copy of anything; a projection model is self-generating: it is its own original. The visual models examined in this chapter are originals in this sense; they differ from the simple maps examined in the previous chapter in their complexity, which is a result of representing staggering amounts of data. The challenge to graphically represent vast amounts of data is met by sophisticated, animated three-dimensional color projection models which can be interactively manipulated. A provocative example, examined below, is the graphical imaging of sheer flow boundary layer turbulence through computer simulation.

3.2. The analysis of turbulence

Turbulence is a very complex set of phenomena which occur in all aspects of nature: the atmosphere, the flow of fluids, convection in a cup of coffee, or smoke from a cigarette. Werner Heisenberg allegedly said on his deathbed that he would have two questions for God: "Why relativity, and why turbulence; I really think he may have an answer to the first question" (Gleick 1987, 121 and 329). Only recently have there been some limited breakthroughs in the physical understanding of the onset and propagation of
turbulence in fluids. The analysis of turbulence aims at gaining an understanding of a widely occurring but extremely complex set of phenomena, which can be partially described by the interaction of a large set of variables.

Turbulence data, whether it comes from measurement or simulation, can be analyzed by statistical analysis; numerical quantitative analysis; or graphical imaging. The statistical methods are often rather crude, since they can only extract a limited amount of very specific information about the system. The use of statistics provides global physics, theories about large numbers of events. René Thom notes that statistical evaluation is blind, and can easily miss morphological shapes of functions made visible in geometric projection (1986, 367).

The graphical representation of data in the form of manipulable visual models is a powerful tool in the study of complicated, multivariate and multidimensional phenomena. Graphical analysis is a progression towards refinement; it allows for the study of particular questions and is an investigation into how individual structures are affected by other structures. Graphical imaging is a process of evolution in scientific methodology occurring in many fields; graphical imaging makes the underlying structure of phenomena and processes visible. It is also controversial, however, since "accidental" correlations can occur because
of the projection method used. Graphic imaging has to be backed up by quantitative or statistical analysis.

One of the challenges of the study of turbulence is to represent the state of the system as a visual model, projected either from measurements or from computer simulation data. The raw data consists of very large sets of numbers, of the order of tens of millions \(10^7\).\(^5\) The computer power necessary to calculate and graphically display projection methods involving variables with millions of numbers and billions of calculations per second (known as gigaflops) has only recently become available. Computers do not just make calculations faster or projections easier. The expansion in the use of computers has redefined the interaction of the researcher with the apparatus. Sometimes, a qualitative change in scientific methodology takes place: this is the case with simulations, and with plotting the results of iterative equations to produce fractal images (see chapter 4). The significant advances made in computer graphical imaging will increase the importance of and reliance on visual models in the study of empirical phenomena (Pickover 1991).

\(^5\)The amount of data that can be processed is determined by the sample chosen, the complexity of the mathematical equations involved, and the processing speed of supercomputers. \(10^7\) is rather modest for this particular study of turbulence; \(10^{10}-10^{16}\) would allow for a much more satisfying analysis (Moquin 1993).
Turbulence data by itself is a meaningless collection of numbers. To a researcher familiar with the techniques used to generate it, and with the physics it represents, the data is not meaningless: in fact, the data contains so much meaning that it has so far proven impossible to completely understand even a small fraction of it (Moquin 1993). In studying visual models produced by graphical imaging, researchers look for correlations, correspondences, or surprising shapes or accumulations of data, and gain an understanding of the interplay of the different variables in turbulence. The different pictures have to be endowed with meaning, and through them, the data has to be endowed with meaning. This in turn should spawn advances in the understanding of the theory.

3.3. Simulation and the construction of mapping algorithms

The data used in the study of boundary layer turbulence produced by sheer flow can be measured from an empirical system, or alternatively, it can be produced by mathematical simulation on a computer. The data studied in this chapter comes from a computer simulation.

It is very difficult to measure an actual flow with great precision, since the measuring devices, in this case laser light, should not interfere with the flow itself. Experiment is limited compared to simulation because even
laser "snapshots" of the flow only provide one or two variables. In experiments, greater reliance on statistics and knowledge of the underlying physics is required. In addition, in all experimental data there is "background noise" caused by the measurement process and outside interference. The advantages of computer simulation data are thus "cleaner" and more complete (or precise) data, without empirical distortions.

The disadvantage of mathematical simulation on a computer is that the simulation is controlled by a mathematical model of the system; the data produced in this fashion is thus also a model of the empirical system. Discoveries about the structure of the data will have to be tested against measurement data later, and so reintegrated into the empirical system.

Computer simulation is a method of modelling which is gradually assuming the status of scientific method. It has been used in modelling numerous phenomena, from interactions between economic agents to the development of lifeforms on earth (Waldrop 1992). A computer simulation consists in setting up mathematical equations and applying them to a chosen domain; usually the application of these mathematical models is very complex or very repetitive. The simulation allows the researcher to control all the laws and parameters of the little universe created on the computer, and thus to investigate in principle all possible behavior of the
system. But the simulation is still a model of the empirical system, and results from simulations must be compared to data from measurement.

Computer simulation is an exciting and rapidly expanding research method, but it is also controversial. Its advantages are that: it is possible to simulate systems which cannot be measured; the simulation is almost always cheaper than experimentation; and the level of control over the data is in principle unlimited. The principal reason for distrusting simulations is that one cannot be certain whether a real-life system is adequately modelled by a simulation (see section 5.3). A simulation is always based on a mathematical model which is a simplification of the system studied and may not be universally applicable. The programmer cannot always adequately judge whether the information put into the system in the form of equation and rules does not prejudice a certain outcome. Because of the complexity of the calculations involved, it may not be possible to check the outcome by any other means. Since the simulation only simulates the results of an experiment, one cannot attain the mathematical rigor of a proof.

There is a formal sense of the word "map" which refers not to the product of a projection function, but to the form and content of the projection method itself. To distinguish such maps from maps that are visual models, we will refer to
them as "mapping algorithms." In the study of turbulence, mapping algorithms are used to turn enormous quantities of numbers into animated, three-dimensional color pictures of specific parts of the system studied. The use of mapping algorithms to produce graphic animation represents an extension of simple topological projection methods.

There are two successive steps in the process of turning numbers into images. The raw data from the simulation of turbulence phenomena has to be produced; then, mapping algorithms are applied to this data to project three-dimensional color images.

To simulate a particular case of boundary layer turbulence, Brian Moquin generates a three-dimensional grid of 128 by 128 by 128 points, yielding a little over two million data points. For each of these points, the velocity of the fluid particle, which consists of three components, is calculated according to the Navier-Stokes equations which govern turbulence. An important philosophical and scientific question is the value and strength of the Navier-Stokes equations as a mathematical model of turbulence; this is the quandary posed by using data from a simulation. There are also problems with the methods of numerical calculation: if the Navier-Stokes equations do not have unique solutions, the whole exercise is in jeopardy.
A number of other variables, such as strain-rate and vorticity, are also calculated directly from the velocity variables and their derivates, in order to be directly accessible when needed. These two million points and their corresponding values represent a simulated snapshot of the system at a given time (Moquin 1993).

The supercomputer stores this vast amount of data; in this case, 80 megabytes per snapshot view of the system. Turning the data into three-dimensional graphical images which the researcher can manipulate is a way of combining an overview of the simulated system with the possibility of literally zooming in to study detailed patterns and dynamical interactions (Moquin 1993). The picture projected from the data is not a single view at a set level, but a visual domain in which the researcher can navigate, focusing on specific areas, contrasting images at different scales. The pictures produced by this graphical imaging technique are dynamic.

3.4. Graphical imaging and animation

In order to manipulate visual images, the data points and the corresponding numerical values of the variables at that point have to be projected into a three-dimensional space. This three-dimensional space in turn must be represented as a two-dimensional image, since, like the page
or the blackboard, the computer screen is flat. Two successive projections are thus required: one first assigns a place in a virtual three-dimensional space to each data point along x-, y- and z- axes; then one uses a number of manipulations, such as tilting and rotation, as well as lighting and shadows, to project a three-dimensional solid on the plane.

Four dimensions of data can be represented by the projection method itself: three geometrical dimensions (reduced to two), and color, which according to convention varies from dark blue for the lowest values to red for peak values. Every data point contains not only the three velocity variables of each fluid particle, but also derived variables such as vorticity (rate of spin), enstrophy (vorticity squared), or pressure, whose value can be plotted at each point.

There are a host of other possible manipulations, such as cutting an isoplane, a two-dimensional cut through a three dimensional space, to visualize the intensity of the variables at a specific level.

A dramatic additional representational projection has been the introduction of animation, veritable dynamic "films" using time as an additional dimension of data. The process can be represented in "real-time," when there is such a notion, or it can be slowed down or speeded up for a better understanding of the state of the system over time.
and the dynamic interplay of variables. Animation need not be seen as an animated image of the development of the system over time; it only means that the parameters guiding one variable are transformed according to a specific function. Time is not one of the variables represented by the graphical imaging procedure explained here, even in animations of the system (Moquin 1993).

Figure 3.2 represents a small section of the three-dimensional grid in the study of homogeneous turbulence. The red tubes represent enstrophy, which is vorticity (rate of spin of the fluid particles) squared. Note the effect of shadows on these tubes, which is vital to achieving the three-dimensional effect. The shadows also show that the tubes are hollow, but this should not be true of the data this image represents; it is an extraneous characteristic of the pictorial manipulations used to draw the tubes. Upon close examination, the edges of the tubes are also not completely smooth: the graphical imaging system (the mapping algorithms) plots tiny triangles between data points in the grid to produce surfaces. This is due to the projection technique, which samples information in discrete values (Moquin 1993).

Figure 3.3 is a small section of the grid in an analysis of boundary layer turbulence. The white cloud represents a certain value of vorticity. The colored shape intersecting with the white cloud is a horizontal isoplane
Figure 3.2. Graphical imaging: enstrophy tubes. Courtesy of Brian Moquin.
Figure 3.3. Graphical imaging: colored isoplane. Courtesy of Brian Moquin.
of velocity values (an isoplane is a two-dimensional cut). The intensity of the variable is initially represented by color, but this effect is doubled up and heightened through a three-dimensional projection, from the highest peak value (red) to the lowest value (purple-blue). The back panel of the figure consists of a vertical isoplane (Moquin 1993).

The images produced can mean radically different things to different people: for a fluid mechanician, they could connote information about the relations between strain-rate and vorticity; from another perspective, for a dynamicist, they could lead to a refinement of ideas as to the topological structure of chaotic regimes (Moquin 1993).

There has been a veritable explosion in the complexity and level of abstractness of projection models. The number of variables, projection methods and manipulations provides an infinite number of possible visual models of the system over time. The system for graphical imaging developed by Moquin constructs visual models interactively: the visual model is "active" rather than "passive" (Moquin 1993). We no longer speak of a single passive image or visual model; we produce a whole combinatory set, a whole matrix of possible images which can be actively manipulated by changing our projection and manipulations parameters.

These visual models are only virtual, visual domains of representation which can be explored with the tools that
helped shape them; hence, they are dynamic visual models. The increase in complexity occurs in a simple dimension; it is the number of variables which allows us to define a dynamic visual model.

Many people argue that graphical imaging is inherently an unreliable mechanism, for it is relatively easy to draw incorrect conclusions based on arbitrary choices in the projection methods. But by controlling the projection methods and manipulations in the visual domain, the researcher gains a better understanding of the influence of graphical imaging methodology on the image, and should thus be able to avoid making false generalizations based on the projection system.

A visual model that can be manipulated, an active image, is far less likely to be unreliable or misleading than a simple, passive image. The active image can be manipulated in many ways, constructed from many perspectives. The visual structure that the researcher takes to represent a correlation between variables can be examined from all these different perspectives. If the structure withstands manipulations, it is most likely not produced as a visual side-effect of the projection methods or manipulation techniques.

One immediate advantage of active images is that because of the control over projections and manipulations, the influence of the projection methods and manipulations is
much more obvious. In order to get any information from graphic imaging, the right questions about the system must be asked; there is simply too much information, even in the images, to look for correlations at random. The challenge is to imbue the endless series of images that can be constructed in this way with enough "meaning" in their representational relation to the system: what aspect of the system does this visual domain represent and what can be noted or concluded from this?

3.5. Projection and virtual modelling: the pi-scape

Gary and Gregory Chudnovsky, emigré mathematicians from the Soviet Union, have calculated pi to a precision of several billion decimal digits (Hall 1992, 259). The big question about pi is whether it is a random number.\(^6\) Pi appears to be a random number; to prove that it is not, some regular pattern or repetition must be found in the billions of digits. This has so far been done by numerical and statistical analysis methods, without conclusive results. A decidedly new approach is to visualize the first million

\(^6\) There are actually two questions: whether pi is random and whether it is normal. There is no technical definition of a random number, but pi appears to be more random than the random sequences generated by random algorithms, since these always have some kind of structure. A normal number is one in which any chosen pattern of numbers will occur at some point; in other words, no numbers are privileged over others. Pi also appears to be normal, although this has not yet been proven (Schroeder 1990, 247).
digits of pi through the concept of a "random walk," where
the differences between successive numbers are calculated
and plotted (Figure 3.4).

But the Chudnovskys are not satisfied with a three-
dimensional pi-scape which has had to be projected back into
two dimensions again; they would like to "move about" in a
three-dimensional virtual reality:

... landscapes [such as the pi-scape] in fact are cheap
imitations of what we really want to do .... A map is
not really a true representation of a landscape,
because for this you really need to see a third
dimension. And that is still done on paper. What I
really want is to be able to assess three-dimensional
information in all directions, and to be able to
reconstruct the depths, and in this way avoid
artificial landscape representation (Gregory
Chudnovsky, in Hall 1992, 260-61).

This is an interesting quote for several reasons: note the
terms used to describe the two-dimensional map of the pi-
scape: "a map is not really a true representation..."; the
"artificial landscape representation" needs to be avoided.
According to our analysis, however, there is nothing more
"real" or "true" about a three-dimensional representation,
especially of something as abstract as a million decimal
digits! It is fascinating to realize the extent to which one
representation can still be preferred over all others, and
that this is justified by an appeal to true representation
of reality.
Figure 3.4. The Chudnovsky Pi-scape. From Hall 1992, color insert.
Amazingly enough, the Chudnovskys' longing for "real" three-dimensional representation may not go unanswered much longer. With the fast-paced development of virtual reality, researchers may soon be able to literally scale the heights of their virtual pi-scape by strapping themselves to a computer. Virtual reality is a computer-defined and computer-generated world; the subject interacts with this world through goggles which project a three-dimensional image in front of the eyes, and through interactive devices such as gloves or sticks. The subject can also be strapped to other devices which simulate the movements undergone in the virtual world.

The emphasis in virtual reality is not just on the visible but also on simulated physical interaction with the virtual world. But the capacity for visualization is the cornerstone for the creation of the illusion of a virtual world. Virtual reality will become the future of projection modelling, allowing for the creation of virtual and dynamic visual models. The researcher is part of the virtual world and will be able to manipulate a new kind of active image by altering the virtual landscape. The creation of visual models will not be limited to the projection of pictures or domains of visualization, but will include the creation of simulated visual worlds. These new methodologies will grow out of the experiments with manipulable visual projection techniques.
3.6. Graphical imaging and representation

The development of superior projection methods through graphical imaging technology has presented scientists with a host of opportunities to visualize the structure of empirical systems which are beyond the horizon of the visible. New projection methods have forced those who construct and use the resulting visual models to provide context and meaning for the pictures they create. An interplay between vast new possibilities but greater and more sophisticated accountability is emerging and transforming the concept of visualization.

Visual projection models are no longer necessarily linked to images: visual models become their own originals. The loss of the visual link and the complexity of the projection methods has made any number of visual models possible. The representational link, however, must always be provided by the creator and users: the picture must be endowed with meaning. Providing the representational link is not just a question of assigning symbols to elements of the visual model in the way a legend functions for a map. Endowing the picture with meaning consists in constructing the projection function by assigning variables to dimensions
or graphic manipulations, and being able to comprehend and interpret the resulting picture.

The structure of the system at the level at which it is being studied cannot be perceived, cannot be naturally associated with images, does not "look like" anything. When one is not dealing with computer simulations, it remains possible to look at the empirical system being studied, since it has a physical presence in the world. But even in this case the visual presence is not exclusively what is being modelled.

Often the variables of the system studied are virtual variables which cannot be detected by perception in any possible experiment, but only calculated. In the case of boundary layer turbulence, spin, vorticity (rate of spin), enstrophy (vorticity squared), and other variables which can be calculated from the initial three velocity variables, cannot be perceived. The graphical images representing the structures do not "look like" the flow itself. They do not look like whorls and eddies that could be perceived in a flow of water. But the information accessible to perception is always limited; in a flow composed only of water molecules, nothing can be seen if one looks closely enough because the flow is mostly empty space. All the data is invisible; only a small part of it is mapped to the screen through the calculation of values and threshold values of fluid particles.
The visual model projected from the interaction of values of virtual variables represents one possible perspective on the structure of these variables; this interaction itself is a virtual state. The visual model produced by its own projection method and manipulations is self-generated. It is not necessarily dependent on, or necessarily related to, any other image or visual model. The visual model represents the empirical system (or the simulation) precisely by being a uniquely constructed perspective not related to perception. It is its own original.

Images are vital for understanding spatial relations between numbers and for discerning patterns that involve more than two numbers. In a graph, only one relationship between numbers is plotted; the advantage of three-dimensional, color graphics is that they make it possible to discern several patterns. In effect, we are adapting our visualization to what our brain can perceive. The fact that the human brain can process three-dimensional color animations with some facility is a result of both the structure of our image-processing apparatus and our cultural understanding of the representational properties of an image. Our capacity to endow constructed pictures with meaning involves our ability to perceive and process pictures, to focus on specific aspects or generalize from a
global perspective, and to see beyond this particular picture.

To interpret a visual model, to define and describe it, is to take a position about the information it contains and illustrates; it is a statement. This statement involves not only the visualization and interpretation skills of the observer, but also deep-seated metaphysical convictions. The Chudnovskys' description of their pi-scape by the words "really," "true" and "artificial" indicates their reluctance to give up a variety of realism, even in the case of an obviously and thoroughly artificial representation. The visual models studied in this chapter illustrate a structure of virtual variables; from a visual point of view, there is nothing "real" or "true" about them.

It is meaningless to ask whether the numerical value of a virtual variable such as enstrophy at a given time within a given system is "real" or not. An informed judgement about the relevance and applicability of a given numerical computer simulation as a model of the natural phenomenon of boundary layer turbulence is of course possible, but this by itself does not make the simulation or any of its numerical properties real. The simulation and a fortiori the visual models are constructed: they may be relevant, exciting, misleading, aesthetically pleasing, informative, and hopefully useful, but never true, and never real.
Above and beyond our natural capacity to process images, we possess an aesthetic and cultural understanding of images. Our ability to imbue an picture with meaning is closely related to our artistic vision, our capacity to appreciate and "make sense" of artistic representation. Symbolists, pointillists, cubists, minimalists, and pop artists have redefined our concept of image, and of representation. We culturally manipulate representation to create our own worldview; I will return to this in chapter 7.